ON PERTURBATION OF THE SPECTRUM OF PLANAR DIELECTRIC WAVEGUIDE BY REFRACTION INDEX PROFILE

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ABSTRACTS

The problem of calculating the eigenwaves of planar dielectric waveguide with arbitrary refraction index profile is considered. The iterative process based on the exact solutions of this problem in the case of piecewise constant profile is investigated. The abstract perturbation theory for operator equation with spectral parameter is extended.

INTRODUCTION

Let the layers of planar dielectric waveguide be separated by the planes $z=0$ and $z=h$. The potential function $F(x,z)=\exp(ik_0\tilde{n}x)f(x)$ of the TE-waves is a solution of the Helmholtz equation and satisfies the conjugation conditions for $z=0$ and $z=h$. The function $f(x)$ is a solution of the boundary value problem

$$f''(z)+k_0^2[n_f^2(z)-\tilde{n}^2]f(z)=0, \quad 0<z<h \ , \tag{1}$$

$$f'(0)-\beta_s f(0)=0, \quad f'(h)+\beta_a f(h)=0, \tag{2}$$

here $\tilde{n}$ is the unknown spectral parameter (longitudinal propagation constant),

$$\beta_a=k_0\sqrt{\tilde{n}^2-n_a^2}, \qquad \beta_s=k_0\sqrt{\tilde{n}^2-n_s^2}\ ,$$

n_s, $n_{a,}$ $n_f(z)$ are the refraction indexes of the substrate, of the external medium and of the waveguided layer respectively. The conditions (2) are to be replaced by other conditions in the case of TM-waves.

If $n_f(z)=const$ then the exact solutions of the problem (1), (2) can be written down in the analytical form. We discuss the possibility of using these solutions for approximate calculating of the solutions of problem (1), (2) in the general case.

ITERATIVE PROCESS

In [1] an iterative method is proposed to solve the following spectral problem

$$f''(z)+C(z,\lambda,p)f(z)=0,\ \ f'(\alpha)+A(\lambda)f(\alpha)=0,\ \ f'(\beta)+B(\lambda)f(\beta)=0\ . \tag{3}$$

Let the solutions of the problem (3) be well-known for $p=0$, and be unknown for $p=1$. We introduce the increasing set of values $p^{(0)}=0,p^{(1)},...,p^{(n)=1}=1$. Let the increments $\delta\lambda$, $\delta f(z)$ correspond to the increment δp. The linearized equations for the calculating of increments have the form

$$\delta f''(z) + C(z,\lambda,p)\delta f(z) = -\left[\frac{\partial C}{\partial \lambda}\delta\lambda + \frac{\partial C}{\partial p}\delta p\right]\left[f(z) + \delta f(z)\right], \tag{4}$$

$$\begin{aligned} \delta f'(\alpha) + A(\lambda)\delta f(\alpha) &= -\frac{\partial A}{\partial \lambda}\delta\lambda\left[f(\alpha) + \delta f(\alpha)\right], \\ \delta f'(\beta) + B(\lambda)\delta f(\beta) &= -\frac{\partial B}{\partial \lambda}\delta\lambda\left[f(\beta) + \delta f(\beta)\right], \end{aligned} \tag{5}$$

$$\delta\lambda\left\{\int_{\alpha}^{\beta} f(z)\frac{\partial C}{\partial \lambda}\left[f(z) + \delta f(z)\right]dz - f(\beta)\frac{\partial B}{\partial \lambda}\left[f(\beta) + \delta f(\beta)\right] + f(\alpha)\frac{\partial A}{\partial \lambda}\left[f(\alpha) + \delta f(\alpha)\right]\right\} =$$

$$= -\delta p\int_{\alpha}^{\beta} f(z)\frac{\partial C}{\partial p}\left[f(z) + \delta f(z)\right]dz . \tag{6}$$

ABSTRACT APPROXIMATE SCHEME

To substantiate the numerical method we consider more general abstract spectral problem.

Let A be a linear operator acting from the space X into X, $\overline{A}$ be a linear operator acting from the space $\overline{X}$ into $\overline{X}$, the space $\overline{X}$ approximate the space X and λ be a complex parameter. The correspondence between the spaces X and $\overline{X}$ is established by the interpolation operator $T : X \to \overline{X}$ and the approximation operator $S : \overline{X} \to X$, here $ST = I$ [2]. Let the exact spectral problem

$$Ax - \lambda x = 0, \qquad x \in X \tag{7}$$

be replaced by approximate spectral problem

$$\overline{A}\overline{x} - \lambda\overline{x} = 0, \qquad \overline{x} \in \overline{X} . \tag{8}$$

We introduce new spectral problem

$$\widetilde{A}x - \lambda x = 0, \quad x \in X, \quad \widetilde{A} = S\overline{A}T. \tag{9}$$

If $\lambda_0, \overline{x}_0$ are the eigenvalue and the eigenelement for the problem (8), then $\lambda_0, \widetilde{x}_0 = Sx_0$ are the eigenpair for the problem (9). Therefore the eigenpair for the problem (8) can be obtained in the form

$$x = \widetilde{x}_0 + \Delta x, \quad \lambda = \lambda_0 + \Delta\lambda$$

with auxiliary normalization condition.

The equation

$$(\widetilde{A} - \lambda_0 I)\Delta x = \Delta\lambda(\widetilde{x}_0 + \Delta x) - \Delta A(\widetilde{x}_0 + \Delta x), \qquad \Delta x \in X \ \mid \ (x, \widetilde{x}_0) = 0 \tag{10}$$

is an abstract analogous of the problem (4), (5),

$$\Delta\lambda = (\Delta A(\tilde{x}_0 + \Delta x), \tilde{x}_0) \ . \tag{11}$$

The operator $\Delta A = A - \tilde{A}$ defines the closeness between the exact and approximate operators [2].

CONVERGENCE CONDITION

It follows from (10), (11) that

$$\Delta x = TP(\Delta x), \tag{12}$$

here T is a linear operator (inverse operator for $\tilde{A} - \lambda_0 I$) and P is a nonlinear operator. It is shown that the equation (12) can be solved by iterative process if the condition

$$\| TP'(\Delta x) \| \leq q < 1 \quad \text{or} \quad 3 \| T \| \, \| \Delta A \| \leq q < 1$$

is fulfilled.

Let $A(\lambda)$ be a linear operator from the space X into X and λ be a complex parameter. Let the exact spectral problem

$$A(\lambda)x - \lambda x = 0, \qquad x \in X \tag{13}$$

be approximated by problem

$$\tilde{A}(\lambda)\tilde{x} - \lambda\tilde{x} = 0, \qquad \tilde{x} \in \tilde{X} \ , \tag{14}$$

here $\tilde{A}(\lambda)$ is a linear operator from $\tilde{X}$ into $\tilde{X}$. In this case the equation (12) is replaced by set of equations

$$\begin{aligned} \Delta\lambda &= \left(\left[A(\lambda + \Delta\lambda) - \tilde{A}(\lambda) \right] (\tilde{x} + \Delta x), \tilde{x} \right), \\ \Delta x &= \left[\tilde{A}(\lambda) - \lambda I \right]^{-1} \left[\Delta\lambda I - A(\lambda + \Delta\lambda) + \tilde{A}(\lambda) \right] (\tilde{x} + \Delta x) \ . \end{aligned} \tag{15}$$

The sufficient condition for the convergence of the iterative process for the system (15) is obtained.

REFERENCES

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